

Light Cone Consistency in Bimetric General Relativity

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Abstract. General relativity can be formally derived as a flat spacetime theory, but the consistency of the resulting curved metric's light cone with the flat metric's null cone has not been adequately considered. If the two are inconsistent, then gravity is not just another field in flat spacetime after all. Here we discuss recent progress in describing the conditions for consistency and prospects for satisfying those conditions.

INTRODUCTION

The formulation and derivation of general relativity using a flat metric tensor $\eta_{\mu\nu}$ are well-known from the works of Rosen, Gupta, Kraichnan, Feynman, Deser, Weinberg *et al.* [1]. One can obtain a curved metric $g_{\mu\nu}$ by adding the gravitational potential $\gamma_{\mu\nu}$ to the flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G}\gamma_{\mu\nu}. \quad (1)$$

This framework is useful [2], but is it merely formal? If general relativity can be *consistently* regarded as a special-relativistic theory, then the observable curved metric must satisfy a nontrivial consistency condition in relation to the unobservable flat background metric: the “causality principle” says that the curved metric's light cone cannot open wider than the flat metric's. The question of the relation between the cones is complicated somewhat by its gauge-variance.

PREVIOUS DISCUSSIONS OF RELATIONSHIP OF NULL CONES

While the flat spacetime field approach to general relativity has been mature since the 1950s, the question of the consistency of the effective curved metric's null cone with the original flat metric's received surprisingly little attention. In the

1970s van Nieuwenhuizen wrote: “The strategy of particle physicists has been to ignore [this problem] for the time being, in the hope that [it] will ultimately be resolved in the final theory. Consequently we will not discuss [it] any further.” [3] More recently (since the late 1970s), this issue has received more sustained attention [4–7], but the treatments to date have been impaired by unnecessarily strict requirements [4,6] or lack of a general and systematic approach [5,7], as we have noted [1].

We propose to *stipulate* that the gauge be fixed in a way that the proper relation obtains, if possible. The gauge fixing can be implemented in an action principle using ineffective constraints, whose constraint forces vanish [8]. This approach does appear to be possible, because the gauge freedom allows one to choose arbitrarily g^{00} and g^{0i} (at least locally). Increasing g^{00} stretches the curved metric’s null cone along the time axis, so that it becomes narrower, while adjusting g^{0i} controls the tilt of the curved null cone relative to the flat one. Stretching alone appears to be enough to satisfy the causality principle, in fact.

KINEMATIC AND DYNAMIC PROGRESS

The metric is a poor variable choice due to the many off-diagonal terms. One would like to diagonalize $g_{\mu\nu}$ and $\eta_{\mu\nu}$ simultaneously by solving the generalized eigenvalue problem $g_{\mu\nu}V^\mu = \Lambda\eta_{\mu\nu}V^\mu$, but in general that is impossible, because there is not a complete set of eigenvectors on account of the minus sign in $\eta_{\mu\nu}$ [9]. There are 4 Segré types for a real symmetric rank 2 tensor with respect to a Lorentzian metric, the several types having different numbers and sorts of eigenvectors [9]. We have recently used this technology to classify $g_{\mu\nu}$ with respect to $\eta_{\mu\nu}$. Two types are forbidden by the causality principle. One type has members that obey the causality principle, but we argue that they can be ignored. The remaining type has 4 real independent orthogonal eigenvectors, as one would hope. In that case, the causality principle is just the requirement that the temporal eigenvalue be no larger than each of the three spatial eigenvalues.

Realizing the condition $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ when the gravitational field is weak, while obeying the causality principle, is nontrivial. The causality principle puts an upper bounding surface on the temporal eigenvalue in terms of the spatial ones, and the surface is *folded*, as seen in 2 spatial dimensions in figure 1. Einstein’s equations have second spatial derivatives of g^{00} (which is closely related to the temporal eigenvalue), so the fold, if not avoided, would imply Dirac delta gravitational ‘forces’ that make the canonical momenta jump discontinuously. On the other hand, avoiding the fold means excluding $g_{\mu\nu} = \eta_{\mu\nu}$! But why fix the temporal eigenvalue in terms of the spatial eigenvalues at the same point only (ultralocally)? It is enough to do so locally, by admitting derivatives. When the derivatives are nonzero, the fold is avoided, but as they vanish, the fold is approached. If such a partial gauge-fixing can be found, then it will facilitate interpreting the Einstein equations as describing a special-relativistic field theory. In such a theory, one would need to consider the

physical situation near the Schwarzschild radius rather carefully.

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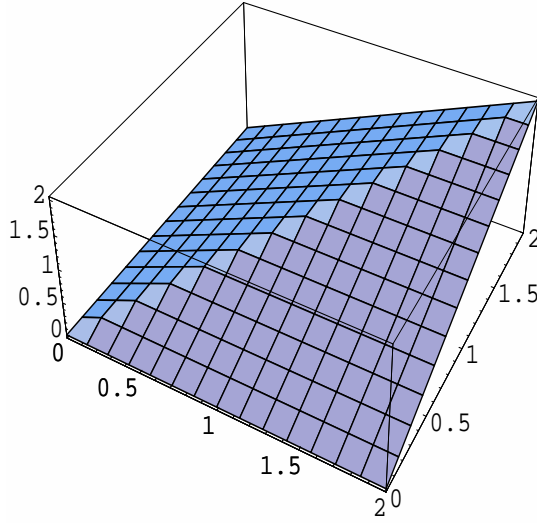


FIGURE 1. Bounding Surface for Temporal Eigenvalue as Function of Spatial Eigenvalues in 2 Dimensions